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MATHEMATICAL MODELS IN MOBILE SENSOR NETWORKS: BALANCE BETWEEN MUTUAL TOPOLOGICAL EQUIVALENCY AND QUALITATIVE-QUANTITATIVE ADEQUACY TO PHYSICAL PHENOMENA

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Математичні моделі в мобільних сенсорних мережах: баланс між взаємною топологічною еквівалентністю і якісно-кількісною адекватністю фізичним явищам

В роботі обговорюється конфлікт між точністю та складністю адекватних математичних моделей наближень, що описують фізичні явища. Сформулюємо задачу керування, коли контрольні функції не можуть безпосередньо діяти на фазові змінні, що визначають заданий термінальний різновид як керуючу мету через праву частину диференціальних рівнянь, що описують вивчену математичну модель. Введено ідею створення ієрархічного каскаду контрольних аттракторів - медіаторів.

The conflict between the accuracy and the complexity of the adequate mathematical modelling approximations describing physical phenomena is discussed. We formulate the control problem when the controlling functions cannot directly act on the phase variables defining a given terminal manifold as a control aim through the right-hand sides of differential equations describing a mathematical model under study. The idea of creating a hierarchical cascade of controlling attractors-mediators is introduced.

The 5-dimensional mathematical model of the longitudinal flight dynamics of a thrust-vectored aircraft in a wing-body coordinate system is governed by [1-7]:

$$\left\{\begin{array}{l} \dfrac{dv}{dt}=\dfrac{P}{m}\cdot\cos\left(\alpha+\delta_p\right)-\dfrac{0.5\rho v^2 S C_x}{m}-g\sin\theta=A_1(\cdot), \\ \dfrac{d\theta}{dt}=\dfrac{P}{mv}\sin\left(\alpha+\delta_p\right)+\dfrac{0.5\rho v^2 S C_y}{m}-\dfrac{g}{v}\cdot\cos\theta=A_2(\cdot), \\ \dfrac{d\alpha}{dt}=q-A_2(\cdot)\Leftrightarrow\left\{\dfrac{d\vartheta}{dt}=q,\alpha=\vartheta-\theta\right\}, \\ \dfrac{dq}{dt}=\dfrac{0.5\rho v^2 S\ell C_m+P\left(y_P+x_P\sin\delta_P\right)}{I_{zz}}, \\ \dfrac{dh}{dt}=v\sin\theta \end{array}\right\} \quad (1)$$

where the expressions for aerodynamic coefficients are

$$\left\{\begin{array}{l} C_Y=C_{Y_\alpha}\sin 2\alpha+C_{Y_{\delta_m}}\delta_m,\quad C_X=C_{X_0}+kC_Y^2, \\ C_m=C_{m_\alpha}\sin 2\alpha+C_{m_{\delta_m}}\delta_m+C_{m_q}\dfrac{l}{v}q \end{array}\right\}. \quad (2)$$

The model of the earth atmosphere is $\rho = \rho(h) = 1.2256\left(1 - 0.2257 \cdot 10^{-4} \cdot h\right)^{4.256}$.

The controls are

- $\delta_m \in \left[-\bar{\delta}_m; \bar{\delta}_m\right]$, aerodynamic control as the angular deflection of an elevator or a movable horizontal stabilizer is at least continuous function of time;
- $\delta_P \in \left[-\bar{\delta}_P; \bar{\delta}_P\right]$, jet engine control as the angular deflection of a turbojet engine nozzle is at least continuous function of time;
- P, aircraft turbojet engine thrust is a controlling parameter taking on constant values within a given time segment.

Here v is airspeed; ρ is atmospheric density; S is wing area; g is gravitational constant; m is mass of the aircraft; h is flight altitude; α is angle of attack; θ is flight path angle; $\{C_Y,\ C_X,\ C_m\}$ are lift, drag and pitching moment aerodynamic coefficients of the aircraft respectfully; I_{zz} is pitch axis aircraft mass moment of inertia; q is pitch attitude angle rate of the aircraft; ϑ is pitch attitude angle; t is time; l is mean aerodynamic chord.

The control aim is to find feedback control laws for δ_m and δ_P that make the aircraft automatically track the given time program of the flight path angle in the form of $\theta = \theta(t) = \theta_m \cdot \left[1 + \sin(\omega t)\right]$ and, at that, the pitch attitude angle must be equal to some given constant value during the entire maneuver, namely $\vartheta = \vartheta'$, regardless of the values of atmospheric density, atmospheric perturbations, aerodynamic coefficients, the value of thrust, aircraft weight, etc. and the initial values of the phase variables $(v,\ \theta,\ \alpha,\ q,\ h)$ within certain given limits.

The design of the tracking control has been done with the assumption that $C_{m_{\delta_m}} \equiv 0$. This means that we use the simplified mathematical model to synthesize the control laws but we will utilize it for the original one (1) with $C_{m_{\delta_m}} \neq 0$. This move definitely makes the problem of the control design much easier to solve. In fact, we cancel only one “small” term, namely $C_{m_{\delta_m}} \delta_m$, in the equations of the original model and the degree of complexity of the problem drops dramatically. But the question arises. How much is this device justified in this specific case from the standpoint of the qualitative and quantitative adequacy of the approximate mathematical models of physical phenomena discussed in the Introduction? The computer simulation shows that the tactic does the trick via the wide-sense robustness provided by the control law that is designed with the help of the Poincaré’s-strategy-based backstepping method. The one ensures, first, mutual adequacy of the original and simplified mathematical models and, second, according to the above-formulated postulate also the adequacy of both the approximate models to the physical phenomenon under study, namely the tracking maneuver of the thrust-vectored aircraft. By mutual adequacy, we mean that the same control applied to both the approximate models of the thrust-vectored aircraft maneuvering produces the topologically equivalent (extended) phase spaces with quantitative differences with imposed limits. After using the Poincaré’s-strategy-based backstepping method, we have obtained the following control laws:

$$\left\{\begin{array}{l} \delta_m = C_{m_{\delta_m}}\left\{\dfrac{1}{0.5\rho v^2 Sl}\left[I_{zz}\left(B_1(\cdot)+B_2(\cdot)\right)-P\left(y_P+x_P\sin\delta_P\right)\right]-B_3(\cdot)\right\}, \\ B_1(\cdot)=a_3\left(q-a_2\left(\alpha-\varphi(\cdot)\right)-A_2(\cdot)-\dfrac{d\varphi(\cdot)}{dt}\right), \\ B_2(\cdot)=a_2\left(\dfrac{d\alpha}{dt}-\dfrac{d\varphi(\cdot)}{dt}\right)+\dfrac{dA_2(\cdot)}{dt}+\dfrac{d^2\varphi(\cdot)}{dt^2}, \quad B_3(\cdot)=C_{m_\alpha}\sin 2\alpha + C_{m_q}\dfrac{l}{v}q \end{array}\right\}, \tag{3}$$

$$\left\{\begin{array}{l} \dfrac{d\delta_P}{dt}=\left\{\dfrac{\partial\varphi(\cdot)}{\partial\delta_P}\right\}^{-1}\left\{D_1(\cdot)+D_2(\cdot)-D_3(\cdot)\right\}, \\ D_1(\cdot)=a_4\left(\vartheta-\hat{\vartheta}\right)-a_2\left(\alpha-\varphi(\cdot)\right)-\dfrac{P}{mv}\sin\left(\alpha+\delta_P\right), \\ D_2(\cdot)=\dfrac{g}{v}\cos\theta-\dfrac{0.5\rho vS\sin 2\alpha}{m}-\dfrac{\partial\varphi(\cdot)}{\partial t}, \\ D_3(\cdot)=\dfrac{\partial\varphi(\cdot)}{\partial v}A_1(\cdot)+\dfrac{\partial\varphi(\cdot)}{\partial h}v\sin\theta+\dfrac{\partial\varphi(\cdot)}{\partial\theta}A_2(\cdot) \end{array}\right\}, \tag{4}$$

where the function $\varphi(\cdot)$ is a solution to the functional equation

$$\left\{\begin{array}{l} \dfrac{P}{mv}\sin\left(\alpha+\delta_P\right)+\dfrac{0.5\rho vSC_Y}{m}-\dfrac{g}{v}\cos\theta-0.175\omega\cos\omega t- \\ -a_1\left[\theta-\theta_m\cdot\left[1+\sin\left(\omega t\right)\right]\right]=0 \end{array}\right\} \tag{5}$$

for the variable α, that is, $\alpha=\varphi\left(t,\omega,v,h,\theta,\delta_P,\delta_m,P,m,a_1\right)$.

These control laws are implemented through a cascade of the controlling attractor-mediators consisting of the two one-codimensional manifolds as follows

$$\left\{\begin{array}{l} M_\alpha=\left\{\begin{array}{l} \dfrac{P}{mv}\sin\left(\alpha+\delta_P\right)+\dfrac{0.5\rho vSC_Y}{m}-\dfrac{g}{v}\cos\theta-0.175\omega\cos\omega t- \\ -a_1\left[\theta-\theta_m\cdot\left[1+\sin\left(\omega t\right)\right]\right]=0\Rightarrow\alpha=\varphi(\cdot) \end{array}\right\}, \\ M_q=\left\{\begin{array}{l} q=a_2\left[\alpha-\varphi(\cdot)\right]+\dfrac{P}{mv}\sin\left(\alpha+\delta_P\right)+\dfrac{0.5\rho vS\sin 2\alpha}{m}- \\ -\dfrac{g}{v}\cos\theta+\dfrac{d\varphi(\cdot)}{dt} \end{array}\right\} \end{array}\right\} \tag{6}$$

It is worth reminding that the engine thrust P of the aircraft is considered a controlling parameter taking on constant values within a given time segment.

The objective of the computer simulation of the original and simplified models of the thrust-vectored aircraft flight is to show the wide-sense robustness and stability in the large, which the synthesized controls laws (3)-(4) provide the ones with. The following values of the model parameters have been used:

$$\begin{Bmatrix} C_{Y_\alpha} = 1, \quad C_{Y_{\delta_m}} = 0.1, \quad C_{X_0} = 0.02, \quad k = 0.1, \quad C_{m_\alpha} = -0.062, \quad C_{m_{\delta_m}} = -0.3, \\ C_{m_q} = -0.75, \quad S = 30, \quad m = 10000, \quad x_p = 0.2, \quad z_p = -0.25, \quad I_{yy} = 40000, \\ l = 5, \quad \theta_m = 0.1, \quad \omega = \dfrac{\pi}{5}, \quad P = 90000. \end{Bmatrix}. \tag{7}$$

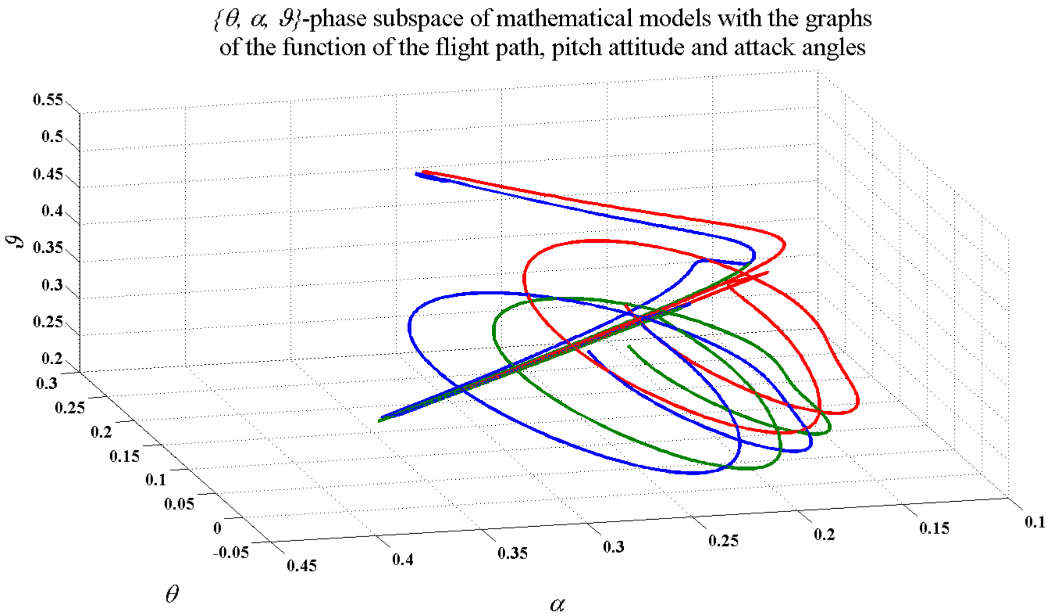

Figure 1. Comparative simulation of the original model versus the simplified one, both governed by the same designed control algorithm: the case of the normal flight simulated by the simplified model is shown in red, the case of the normal flight simulated by the original model is in green, the case of the troubled flight simulated by the original model is shown in blue.

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