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DISPERSION MANAGEMENT IN SINGLE-MODE FIBERS

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1. Introduction

Transfer of digital data packets in optical fibers is performed by propagation of digital signals at high bit rates. In reality Gaussian shaped pulses of duration τ_B are used. Bit rate B relates to the bit sequence or time distance Δt_B between neighboring bits. For example, a bit rate of B = 1 Gbps results in $\Delta t_B = 1/B = 10^{-9}$ s = 1000 ps. In accordance with the recommendations of ITU-T a high-quality transmission can be achieved only if the bit prolongation during propagation in the fiber is no more than $\Delta t_B/10$; it means in case of 1 Gbps transmission bit duration should be no longer than 100 ps. Note that at very high bit rates (B ≥ 1 Gbps) we find only Gaussian shaped pulse in the fiber. In the following we will consider the reasons of pulse prolongation. We will present studies on the influence of chromatic dispersion (CD) as well as polarization mode dispersion (PMD) to the bit rate and its influence on the maximum transmission length in Standard Single-Mode Fibers (SSMF). Nature of CD and PMD is described elsewhere [1].

2. Basics for calculations

2.1. Standard single-mode fibers (SSMF)

We used standard single-mode fibers for calculations with the following set of parameters:

- Core diameter d = 8.5 µm
- Core material: 96% SiO_2 + 4% GeO_2, the refractive index is $n_2(\lambda) = n_{core}(\lambda)$.
- Cladding material: 98% SiO_2 + 1% GeO_2, the refractive index is $n_1(\lambda) = n_{clad}(\lambda) < n_2(\lambda)$.

Based on 3-term Sellmeier equation [2] for these core and cladding materials one can calculated the wavelength dependence of core (n_2) and cladding indexes (n_1) as well as its derivatives and the corresponding group indexes (n_{g2} and n_{g1}).

2.2. Dispersion in SSMF

The following kinds of dispersion are discussed:

- *Material dispersion* results from the line width (spectral broadening) of the light source, here in semiconductor lasers. Material dispersion parameter D_{Mat} can be calculated by $D_{Mat} = -\frac{\lambda}{c_0}\frac{d^2n}{d\lambda^2}$, where c_0 is the speed of light. Material dispersion parameter is mostly identical for core and cladding.

- *Waveguide dispersion* considers the influence of field strength in core and cladding. Thus we get an effective index n_{eff} and an effective group index $n_{g,eff}$. A waveguide dispersion parameter D_{WG} is used.

- *Chromatic dispersion* is the sum of material and waveguide dispersions. The corresponding parameter D_{Chrom} is given by

$$D_{Chrom} = \frac{1}{c_0}\frac{dn_{g,eff}}{d\lambda} = -\frac{\lambda}{c_0}\frac{d^2n_{eff}}{d\lambda^2} = D_{Mat} + D_{WG}$$

Polarization dispersion results from different propagation speed of light with different polarization. In this case one gets instead of a pulse prolongation a splitting of the pulse.

2.3. Power spectrum of bits

To perform simulation, we use the spectral shape of light at high bit rates B. Spectral shape (power spectrum) P(λ) of with Gaussian shaped laser pulses (= bits) with the central wavelength λ_0 and $\Delta\lambda$ the line width. Minimum line width of about $\Delta\lambda = 10^{-4}$ nm at about λ_0 = 1500 nm can be found in commercially used Distributed FeedBack (DFB) lasers. Now we assume that the carrier frequency f_L is modulated with a modulation frequency Δf_M. Mathematically modulation of light waves with modulation frequency Δf_M can be described as modulation of the carrier frequency f_L. It results in generation of side-bands $f_L \pm \Delta f_M$. The modulation frequency Δf_M (e.g. GHz) roughly corresponds to the bit rate B (e.g. Gbps), i.e. $\Delta f_M \approx B$; than we get the total line width as:

$$\Delta\lambda_{total} = \Delta\lambda_L + \left|\frac{\lambda_L^2}{c_0}\right| \cdot \Delta f_M \approx \Delta\lambda_L + \left|\frac{\lambda_L^2}{c_0}\right| \cdot B$$

If $\Delta\lambda_L >> |\lambda_L^2/c_0| \cdot B$ the laser line width dominates; otherwise (at $\Delta\lambda_L << |\lambda_L^2/c_0| \cdot B$) modulation dominates and one gets $\Delta\lambda_{total} \approx |\lambda_L^2/c_0| \cdot B$. Calculations of total laser line width are presented in Tab. **1**.

Tab. 1 Estimations of total line width (at λ_L = 1.5 µm)

$\Delta\lambda_L$ (nm)	1	10^{-4}	10^{-4}	10^{-4}
B (Gbps)	1,10 or 40	1	10	40
$\Delta\lambda_{total}$ (nm)	1	0.0076	0.076	0.300

3. Bit prolongation by chromatic dispersion

3.1. Standard calculations

The often used way is to assume a rectangular spectral shape with a line width $\Delta\lambda_{total}$ and to calculate bit prolongation per kilometer fiber length. To this end we can use the product of dispersion parameter D_{Chrom} and spectral line width $\Delta\lambda_{total}$:

$$\frac{\Delta\tau_B}{L} = D_{Chrom} \cdot \Delta\lambda_{total}.$$

For example, with $\Delta\lambda_{total}$ = 0.0076 nm (for B = 1 Gbps, see Table) and $D_{Chrom} = 16 \frac{ps}{km \cdot nm}$ we get:

$$\frac{\Delta\tau_B}{L} = 0.12 \frac{ps}{km}.$$

That means the maximum permitted bit duration τ_{max} = 100 ps will be reached after L = 822 km.

3.2. Exact calculation

For exact calculations it is much better to use a Gaussian spectral shape. For modelling we assume that power spectrum consists of partial wave groups with slightly different wavelength λ_k. The wavelength difference between neighboring wave groups is $\Delta = \lambda_k - \lambda_{k-1}$, each wave group has identical line width $\Delta\lambda_k << \Delta\lambda_{total}$. All partial wave groups together cover the spectrum *inside* the Gaussian envelope. For each partial wave we can assume again a Gaussian shape with the line width $\Delta\lambda_k$, where k denotes the "number" of wave group in the interval –k … 0 … +k. The term e^{-k^2} describes a Gaussian shaped envelope of wave group number k. Selecting k as an odd number with $\Delta = \lambda_k - \lambda_{k-1} = \frac{\Delta\lambda_{total}}{k+1}$ one can cover the line width of the spectrum.

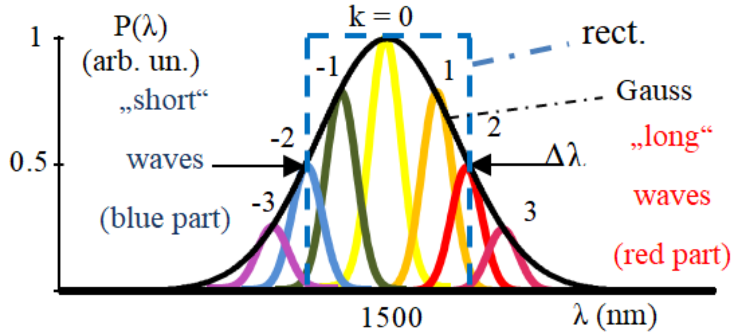

Fig. 1 Rectangular (rect) and Gaussian spectrum (Gauss) with partial waves ("rainbow colors")

For illustration we selected k = 3, i.e. k = -3 … 0 … +3. As one can see in Fig. 1 we have 2k+1 = 7 partial waves. If the total line width is $\Delta\lambda_{total}$ = 0.0076 nm (i.e. for 1 Gbps bit rate) the distance of two neighboring partial wave groups is $\Delta = \frac{\Delta\lambda_{total}}{k+1}$ = 0.0019 nm. The line widths of each partial wavelengths in Fig. 1 is $\Delta\lambda_k = \frac{\Delta\lambda}{2k+1} = 0.00109 \ nm$ i.e. the whole spectrum is covered by these 2k+1 partial wave groups. Because we find in Fig. 1the "rainbow colors" from violet via blue, green, yellow, orange till red – that is why often the terms "blue part" (for shorter wavelengths) and "red part" (for longer wavelengths) are used.

Modelling of bits prolonged by chromatic dispersion means we have to consider the *temporal shape* (time-dependent power) P (t) of laser pulses (= bits) which is given by a Gaussian shaped envelope with bit duration τ_B and central time t_0. Now we will use this model to describe bit prolongation by chromatic dispersion. Without any influences (e.g. dispersion) *all* Gaussian spectral shaped partial wavelengths λ_k coincide in time (it corresponds to the situation at the beginning of the fiber, L = 0). It means all wavelengths contribute to the pulse with a certain weight. Partial waves propagate with different group indexes ($n_{g,eff}^{blue} < n_{g,eff}^{red}$) and group speeds – blue part propagates faster than red part $\left(v_{g,eff}^{blue} > v_{g,eff}^{red}\right)$. That means after a certain fiber length we can expect a picture like sketched in the next Figure (b) with time duration of envelope $\tau_L > \tau_B$; the temporal shift of partial waves results in a broadening of the bit (Figure (c)). Note that in real calculations we used much higher values of k (typically k = 499). For a Standard Single-Mode Fibers (SSMF) we used $D_{Chrom} = 15.9 \frac{ps}{km \cdot nm}$ (at λ = 1.5 μm). At the same conditions as before ($\Delta\lambda_{total}$ = 0.0076 nm and k = 499) we get $\Delta = \frac{\Delta\lambda_{total}}{k+1} = 1.52 \cdot 10^{-5} \ nm$ the time delay between partial wavelengths as $\frac{\Delta\tau_B}{L} = D_{Chrom} \cdot \Delta = 2.42 \cdot 10^{-4} \frac{ps}{km}$. Now one can perform simulation of chromatic dispersion and corresponding limits of transmission length. One has to know the following parameters (with examples):

- Central wavelength λ_0 = 1500 nm and line width Δλ = 0.0076, 0.076 or 0.304 nm (corresponding to a modulation of B = 1, 10 or 40 Gbps, see Tab. 1);
- Chromatic dispersion parameter is D_{Chrom} = 15.9 ps/km·nm;
- k_0 = 499, number of partial wave group ($-k_0$ … -1,0, +1 … k_0) resulting in $2k_0+1$ partial wave groups;
- initial bit duration τ_0 = 10, 5 or 1 ps;
- fiber length L.

Results of calculations are presented in Fig. **2**. The prolongation of the initially 10 ps bit is sketched for increasing fiber lengths L. At L = 440 km we get a 100 ps (FWHM) bit. That means the limit for a high-quality transmission at 1 Gbps bit rate is 440 km.

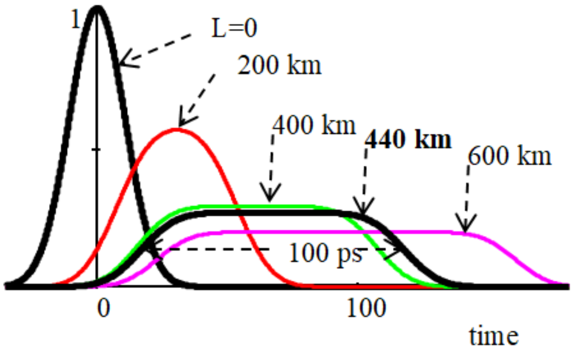

Fig. 2 Bit prolongation by chromatic dispersion for increasing fiber lengths L

The same procedure was performed at higher bit rates. For B = 10 Gbps the initial bit duration τ_0 = 5 ps is prolonged till the maximum permitted duration τ_{max} = 10 ps at fiber length L = 4.4 km; for B = 40 Gbps prolongation from τ_0 = 1 ps to τ_{max} = 2.5 ps we get at L = 0.28 km.

Calculation results of chromatic dispersion limited fiber length for high-quality transmission in spectrum with rectangular and Gaussian spectral shapes are presented in

Tab. **2**. Considering real spectral shape (Gaussian) of modulated laser light we obtained shorter fiber length for high-quality transmission.

Tab. 2 Comparison of maximum fiber length for high-quality transmission in spectrum with rectangular and Gaussian spectral shapes

Spectral shape	L_{max} (km) at B = 1 Gbps	L_{max} (km) at B = 10 Gbps	L_{max} (km) at B = 40 Gbps
Rectangular	822	8.2	0.52
Gaussian	440	4.4	0.28

For dense wavelength division multiplexing (DWDM) one can transfer the bitrate 40 Gbps in 40 channels with 1 Gbps per channel over 440 km fiber length, which is much better than transfer of 40 Gbps in one single channel over 280 m only.

4. Modelling of polarization mode dispersion

Polarization mode dispersion becomes important if we can compensate chromatic dispersion in the fiber, e.g. by a dispersion-compensating fiber (DCF).

4.1. Standard calculation

Like for chromatic dispersion one can calculate maximum transmission length L_{max} assuming a rectangular time shape with a bit duration τ_{max} given by the bit rate B $\left(\tau_{max} = \frac{1}{10 \cdot B}\right)$. To this end we can use the product of dispersion parameter D_{PMD} and square root of maximum fiber length $\sqrt{L_{max}}$:

$$\Delta\tau_{max} = D_{PMD} \cdot \sqrt{L_{max}} = \frac{1}{10 \cdot B}.$$

For example, for B = 10 Gbps and $D_{PMD} = 0.103 \frac{ps}{\sqrt{km}}$ we get:

$$L_{max} = \frac{1}{100 \cdot B^2 \cdot D_{PMD}^2} = 10000\text{km}.$$

That means the maximum permitted bit duration τ_{max} = 10 ps will be reached after L = 10000 km.

4.2. Exact calculation

For PMD one has to calculate the superposition of *two* orthogonally polarized parts of the bit. With increasing fiber length L one can find prolongation and even "splitting in time" of these two differently polarized bits (

Fig. 3). With the measured value of D_{PMD} = 0.103 ps/√km after L = 215 km the initial τ_0 = 1 ps bit (L = 0 km) is splited into two bits with a total bit duration of τ_{215} = 2.5 ps. This is the maximum bit duration permitted at B = 40 GBps bit rate (corresponding to a time between two neighboring bits of Δt_B = 1/B = 25 ps and a maximum permitted bit prolongation of $\Delta t_B/10 = \tau_{max} = \tau_{215}$ = 2.5 ps).

In this case we get the following picture of two neighboring bits in a 40 Gbps transmission line (

Fig. 3). With D_{PMD} = 0.103 ps/√km one gets τ_{215} = 2.5 ps after 215 km.

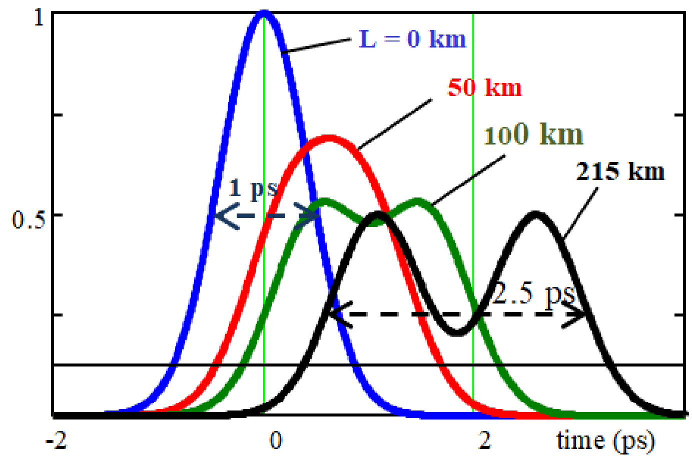

Fig. 3. Influence of PMD after fiber length L for D_{PMD} = 0.103 ps/√km

With increasing bit rate to be transferred one gets a decreasing maximum fiber length. We performed similar calculations as before for other bit rates and found the following:

- For B = 10 Gbps (Δt_B = 100 ps) a bit prolongation from 5 ps to 10 ps is reached after 2700 km fiber length;
- For 40 Gbps (Δt_B = 25 ps) bit prolongation from 1 ps to 2.5 ps is reached after a fiber length of 240 km
- For 80 Gbps (Δt_B = 12.5 ps) bit prolongation from 0.5 ps to 1.25 ps is reached after a fiber length of 52 km
- PMD limited transmission length depends on the duration of initial bits: The lower the initial bit duration the longer is the maximum transmission length. In case of $\tau_0 \to 0$ we get the values of rectangular pulse shape.

Results of calculations of PMD-limited transmission length are summarized in Tab. **3**.

Tab. 3 Comparison of maximum fiber length for high-quality transmission with rectangular and Gaussian temporal shapes

Time shape	L_{max} (km) at B = 10 Gbps	L_{max} (km) at B = 40 Gbps	L_{max} (km) at B = 80 Gbps
Rectangular	10000	625	156
Initial bit duration τ_0 (ps)	5	1	0.5
Gaussian	2700	215	52

References

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