

## ANALYSIS OF THE MAIN CHARACTERISTICS OF MTIE FUNCTION

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### Исследование основных свойств функции МТИЕ

Рассмотрены основные свойства функции максимальной ошибки временного интервала МОВИ (МТИЕ) и возможности использования этих свойств для решения практических задач оценки качества сигналов в современных телекоммуникационных сетях. Проанализированы различные подходы к оценке МОВИ.

The research of accuracy and stability metrics is an actual issue now. The traditional approaches to evaluating and measuring of synchronization signals metrics should be adopted to the new packet networks, e.g., a Maximum Time Interval Error (MTIE) function. The MTIE is one of the main time-quantities for clock stability specification in telecommunications standards. The MTIE is a function of TIE (Time Interval Error) that is evaluated by the samples  $\{x_i=x(i\tau_0), i=1,2,\dots, N\}$  of the difference between a given timing signal and a reference signal within the measurement period ( $T=N\tau_0$ ), where  $\tau_0$  is the sampling interval and  $N$  is a sequence of equally spaced samples of  $x(t)$ . In general case, the  $TIE(\tau_0)$  is a function of systematic (i.e. initial frequency offset  $y_0$  and frequency drift  $D$ ) and random (i.e. phase/frequency modulated timing signal by white/flicker noise etc.) variables. We can add here measurement error (measurement uncertainty) and miscalculation, round-off error. So, in general case,  $TIE \equiv x_i$  is no stationary (time-dependent) random variable.

MTIE is the maximum peak-to-peak variation of the TIEs ( $x_i$ ) within an observation time ( $\tau=n\tau_0$ ) for all observation times. In practice MTIE is estimated by the following formula [1]:

$$MTIE(n\tau_0) = \max_{1 \leq k \leq N-n} [\max_{k \leq i \leq k+n} x_i - \min_{k \leq i \leq k+n} x_i], \quad n = 1, 2, \dots, N-1, \quad (1)$$

where  $x_i$  and  $N$  are stated above,  $k$  – sequence number of the observation interval start and  $n$  – the number of samples at the  $k$ -th observation interval  $\tau_k = n\tau_0$ . If the  $x_{ppk}$  is the peak-to-peak  $x_i$  within  $k$ -th observation, that is  $x_{ppk}(\tau_k) = [\max_{k \leq i \leq k+n} x_i - \min_{k \leq i \leq k+n} x_i]$ , then  $MTIE(\tau)$  is the maximum  $x_{ppk}(\tau_k)$  for all observation intervals of length within  $T$  and (1) may be presented in the shorted form:

$$MTIE(n\tau_0) = \max_{1 \leq k \leq N-n} [x_{ppk}(\tau_k)], \quad n = 1, 2, \dots, N-1 \quad (2)$$

Note that  $x_{pp} = TIE_{pp}(\tau)$  and  $MTIE(\tau)$  is the maximum  $x_{pp}$  ( $TIE_{pp}(\tau)$ ) for all observations of length  $\tau$  within  $T$  [1]. The expressions (1) and (2) can be interpreted as well-known “sliding window” method for the review of TIE data files. Because of

its peak-to-peak nature,  $MTIE(\tau)$  specifications are well suited to support the design of telecommunication equipment buffer size [1, 2].

The  $MTIE(\tau)$  features have been investigated several times. In particular, there is an approach which attempts to take into account MTIE dependence on measurement period  $T$  by studying the statistical properties of the peak-to-peak phase fluctuations based on the  $\beta$ -percentile MTIE concept [3, 4]. As a result of this research activity, ITU-T Recommendation G.810 [1] has redefined MTIE as a specified percentile  $\beta$  of the  $M$  sets ensemble  $\{X_1, X_2, \dots, X_M\}$  of measured data if measurements are made for multiple measurement periods  $M$ :

$$X_i = \max_{\substack{1 \leq k \leq N-n \\ k \leq i \leq k+n_i}} [x_{ppi}], \quad i = 1, 2, \dots, M, \quad (3)$$

while the estimator formula (1) has been left unchangeable. So, it was supposed that  $\beta$ -percentile MTIE concept allowed to estimate percentile MTIE values basing on the standard deviation of the underlying noise (in particular, under the assumption of Gaussian WPM noise). But applying the classical stationary process theory to MTIE estimation is quite questionable because MTIE is not a pure random function but rather a result of some processing of random (no stationary, in general case) variables obtained during the measurement or modeling. If measurement errors have Gaussian distribution, their upper and lower limits can be easily obtained from the Gaussian distribution percentiles. But these issues are rather in the scope of metrology.

The another approach to MTIE analysis is based on the general features of  $MTIE(\tau)$  function and use these features for identification of phase distortions deterministic components (initial offset  $y_0$ , line drift  $D$  and periodical components) on the  $MTIE(\tau)$  behavior [4, 5, 6]. The general features of MTIE function directly resulted from  $MTIE(\tau)$  general definition (1) are the following:

- a) an analog discrete time function of the continuous random sequence  $x_i$ ;
- b) a non-negative function ( $> 0$ );
- c) a non-decreasing function;
- d) a concave function:
  - if for  $k$ -th observation interval  $\tau$  the frequency offset  $\pm y_0$  predominates than:
    - $MTIE(\tau)$  is linearly increasing (irrespective of the offset sign), behaves as  $k\tau_0$ , and has the same slope as the original TIE function;
    - $MTIE(\tau)$  is a line function (so, is concave and convex),
- e) in the presence of line frequency drift (ageing) in set of  $TIE(\tau_0)$  the MTIE values are monotone increasing  $\sim(-\tau^2/2 + N \cdot \tau_0) \cdot D$  for all  $\tau \leq N \cdot \tau_0$  when “window” width  $\tau$  (or  $n$ ) is increasing and reach their maximum under  $N \cdot \tau_0$  (or  $n = N$ );
- f) the MTIE processing converts any convex function (convex data set) of  $TIE(\tau_0)$  into the concave function  $MTIE(\tau)$ ;
- g) joining of sets  $MTIE_1(TIE_1) + MTIE_2(TIE_2) + \dots + MTIE_n(TIE_n)$  equals MTIE of set sum  $TIE_1 + TIE_2 + \dots + TIE_n$ ;
- h) if there is phase jump  $\Delta x \leq \tau_0$ , then  $MTIE(\tau)$  gets up for all  $\tau \geq \tau_0$ ;

- i) if there is phase modulation of TIE by some periodical process only (with amplitude  $A$  and period  $T_p$ ), then  $MTIE(\tau)$  gets maximum pick-to-pick value  $x_{pp} = 2A$  in the observation interval  $\tau \cong T_p/2$ , and stays constant for all  $\tau \geq T_p/2$ ;

In addition, the  $MTIE(\tau)$  because of mentioned above (it appears from the feature “c”) has some features inherent in cumulative function after normalization to the global maximum MTIE value. For instance, the presentation of MTIE in the normalized form allows comparing the different MTIE functions behavior.

MTIE calculation process may be shortened as it results from the listed above features. Specifically, the feature “g” permits to split TIE set into systematic and random parts for calculation and analysis and than combine them into complete full  $MTIE(\tau)$ . Such an approach can be applied to the new stability metrics calculation in packet network environment (for example, in packet selection and filtering process).

There are a lot of studies on this field. For example, in [7] the new metrics based on TIE are proposed for evaluation precise time messages using two-way network protocols (specifically, Precise Time Protocol – PTP). It is proposed to use the maximum likelihood estimation statistics to estimate TIE-based metrics. The close correlation between the new metrics for packet networks (e.g., MATIE, MAFE) and MTIE has been found [8], so the research of this phenomena will go on subject to the listed above MTIE general features.

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