

# SCATTERING OF PLANE WAVES ON PSEUDO-ROTABLE LATTICES OF CYLINDRICAL DIELECTRIC RESONATORS

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## РОЗСІЮВАННЯ ПЛАСКИХ ЕЛЕКТРОМАГНІТНИХ ХВИЛЬ ПСЕВДООБЕРТАЛЬНИМИ РЕШІТКАМИ ЦИЛІНДРИЧНИХ ДІЕЛЕКТРИЧНИХ РЕЗОНАТОРІВ

Розглядається задача розсіювання плоских електромагнітних хвиль на решітках псевдообертаних діелектричних резонаторів (ДР) циліндричної форми з нижчими коливаннями магнітного типу. Аналізуються особливі випадки розсіювання, коли вісі резонаторів колінеарні з вектором розповсюдження падаючої хвилі, або направлені у напрямку відбитої хвилі.

Various periodic structures with Dielectric Resonators (DR) enhance the control of the scattered field [1-2, 4].

Pseudo-rotations of dielectric resonators with no degenerate oscillations in lattices can also lead to a number of interesting cases, determined by the degree of their coupling with the incident and scattered waves. By pseudo-rotation we mean a nontrivial rotation of the axes of the DR in which the relative spatial orientation of the resonators remains constant.

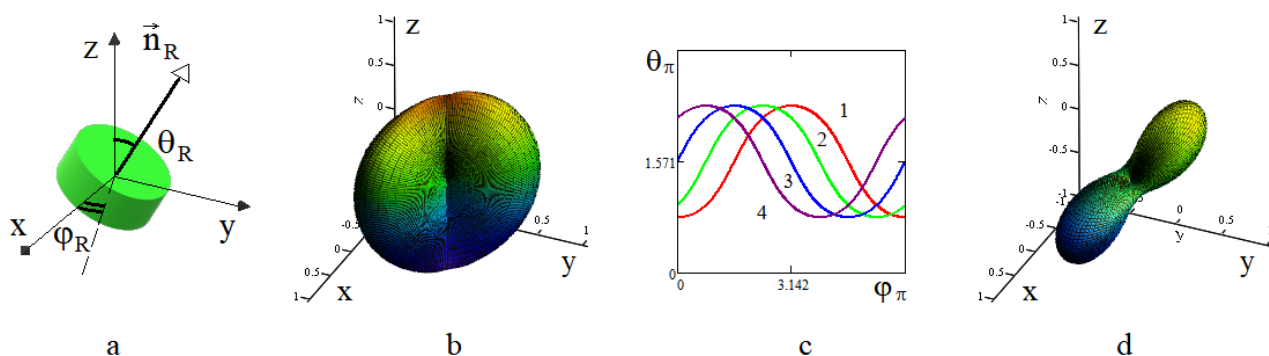


Fig. 1. Rotation of a cylindrical DR relative to a given coordinate system  $(x, y, z)$  (a).

Dependences of the c-functions of the magnetic oscillation  $H_{108}$  of a cylindrical DR on the directions of incidence of a plane wave in spherical coordinates for p-scattering (b):  $\vartheta_R = \pi/4$ ;  $\varphi_R = \pi/4$  for s-scattering (d):  $\vartheta_R = \pi/4$ ;  $\varphi_R = 0$ . Dependences of zero values of c-functions on the angles of incidence of a plane wave in the case of s-scattering for  $\vartheta_R = \pi/4$ ;  $\varphi_R = 0$  (curve 1 (c));  $\vartheta_R = \pi/4$ ;  $\varphi_R = \pi/4$  (curve 2 (c));  $\vartheta_R = \pi/4$ ;  $\varphi_R = \pi/2$  (curve 3 (c));  $\vartheta_R = \pi/4$ ;  $\varphi_R = 3\pi/4$  (curve 4 (c)).

The first special case of scattering occurs when the conditions are satisfied under which all resonators of the array are not coupled with the incident wave. Then the dielectric structure interacts with the incident field only in a nonresonant manner, which leads to the effect of quasi-complete transmission of the field through the grating. A similar effect was first observed by Malyuzhinets for scattering dielectric and metal bars by gratings [3 - 5].

The second special case of scattering occurs when the axes of the resonators are oriented in space in such a way that the direction of the “zero” field in the wave zone of each of them coincides with the “angles of reflection” of the incident wave.

Both of these cases are determined by the properties of the so-called c-function, which determines the relationship between the field of a plane wave and the field of a dielectric resonator of a given orientation [7].

For the main natural oscillations of cylindrical DRs  $H_{10\delta}$ , we have found a general analytical expression for the c-function in the case of rotating resonators:

$$c_i^+ = -2\pi i E_0^* \cdot \frac{h_1}{\beta} \cdot \left(r_0^2 \frac{L}{2}\right) \cdot (k_1^2 - k_0^2) \frac{(\vec{n}_H, \vec{n}_R)}{\sqrt{1 - (\vec{n}_k, \vec{n}_R)^2}} \cdot \frac{[q_\perp \sqrt{1 - (\vec{n}_k, \vec{n}_R)^2} J_0(q_\perp \sqrt{1 - (\vec{n}_k, \vec{n}_R)^2}) J_1(p_\perp) - p_\perp J_0(p_\perp) J_1(q_\perp \sqrt{1 - (\vec{n}_k, \vec{n}_R)^2})]}{(q_\perp \sqrt{1 - (\vec{n}_k, \vec{n}_R)^2})^2 - p_\perp^2} \cdot \frac{[p_z \sin p_z \cos(q_z(\vec{n}_k, \vec{n}_R)) - q_z(\vec{n}_k, \vec{n}_R) \cos p_z \sin(q_z(\vec{n}_k, \vec{n}_R))]}{(q_z(\vec{n}_k, \vec{n}_R))^2 - p_z^2}. \quad (1)$$

Where  $\vec{n}_H$ ,  $\vec{n}_k = (\sin \vartheta_\pi \cos \varphi_\pi, \sin \vartheta_\pi \sin \varphi_\pi, \cos \vartheta_\pi)$ ,  $\vec{n}_R = (\sin \vartheta_R \cos \varphi_R, \sin \vartheta_R \sin \varphi_R, \cos \vartheta_R)$  - is the unit vectors determining the direction of the magnetic field, the direction of propagation of the incident wave, and the axis of the DR in a spherical coordinate system (fig. 1, a), respectively. Here  $p_\perp = \beta r_0$ ;  $p_z = \beta_z L / 2$ ;  $\beta$ ,  $\beta_z$  - transverse and longitudinal wave numbers, which determine the field of natural oscillations of the DR [7],  $q_\perp = k_0 r_0$ ;  $q_z = k_0 L / 2$ ;  $r_0$  - radius;  $L$  - height DR;  $k_0 = \omega / c$ ;  $k_1 = k_0 \sqrt{\varepsilon_{1r}}$ ;  $\varepsilon_{1r}$  - relative dielectric constant of the resonator.

As follows from (1), in the case

p-scattering dielectric resonators of the lattice are not coupled with the incident wave ( $c_i^+ = 0$ ) if

$$\varphi_\pi = \varphi_R \quad (2)$$

For s-scattering

$$\text{tg}(\vartheta_R) \text{ctg} \vartheta_\pi = \cos(\varphi_R + \varphi_\pi). \quad (3)$$

or, in a particular case, if

$$\varphi_\pi = -\varphi_R \quad \text{and} \quad \vartheta_\pi = \vartheta_R; \quad (4)$$

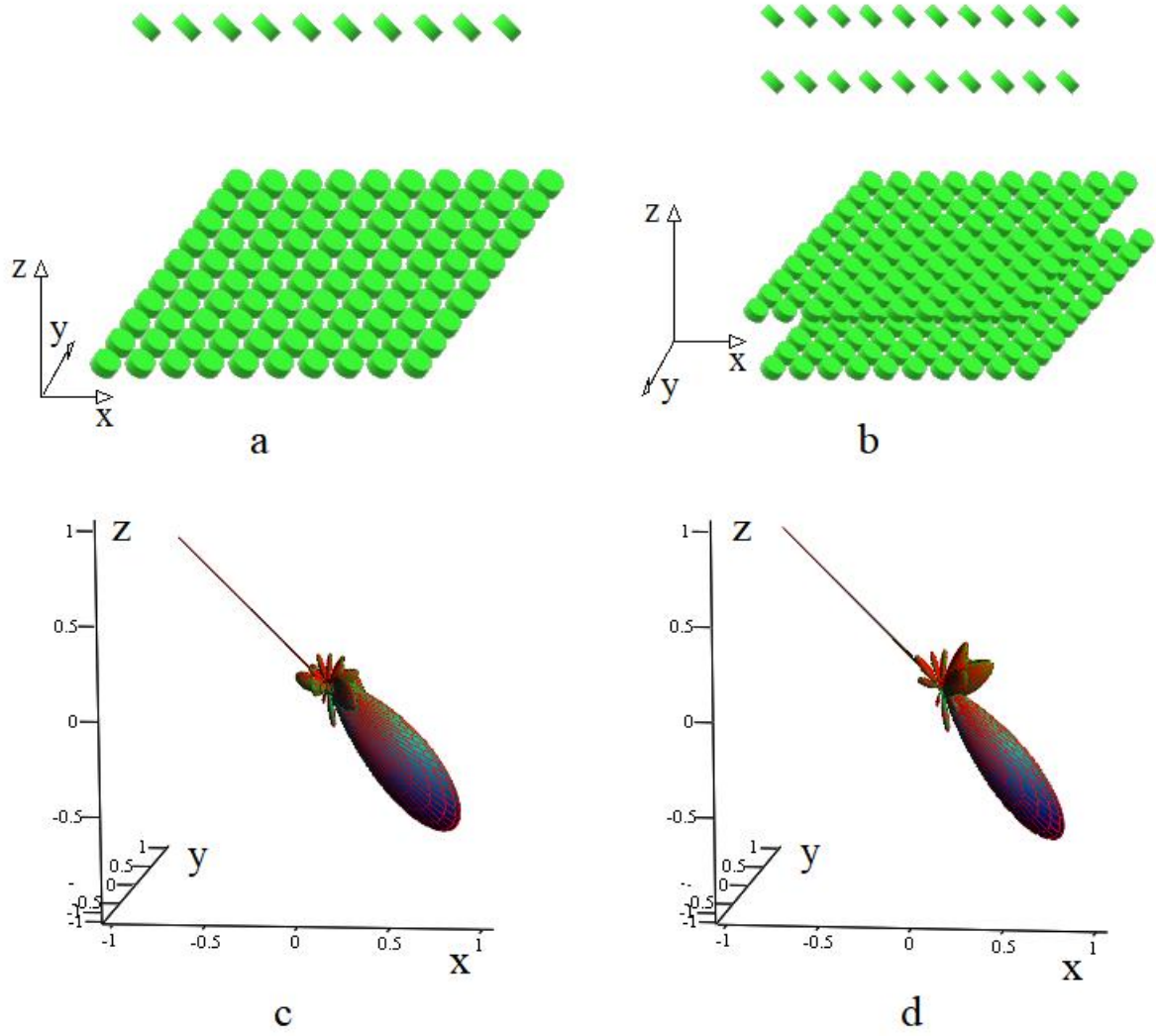


Fig. 2. Square lattices of various orientations of pseudo-rotatable cylindrical DRs - a - c. Angular dependences of the modulus of the scattering amplitude  $|f(\theta_\pi, \varphi_\pi)|$  for  $\vartheta_\pi = 3\pi/4$ ;  $\varphi_\pi = \pi$  and  $\vartheta_R = \pi/4$ ;  $\varphi_R = 0$  - (d) for s-scattering. (Straight lines show the direction of the incident wave).

The numerical solution of equation (3) is shown in Fig. 1c, for different directions of the DR axis; the dependencies  $|c_t^+|^2$  for p and s scattering on the angles of incidence  $(\vartheta_\pi, \varphi_\pi)$  for  $(\vartheta_R, \varphi_R) = (\pi/4, \pi/4)$  are shown in fig. 1, b, d respectively.

Coupling reaches its maximum in case p-scattering, if

$$\theta_R = \pi/2 \text{ and } \varphi_\pi = \varphi_R \pm \pi/2; \quad (5)$$

and for s-scattering:

$$\varphi_\pi + \varphi_R = 0; 2\pi \text{ and } \theta_R - \theta_\pi = \pi/2, \quad (6)$$

or

$$\varphi_\pi + \varphi_R = \pi \text{ and } \theta_R + \theta_\pi = \pi/2. \quad (7)$$

Calculation of the scattering amplitude in the wave zone of one- and two-layer lattices in the case of coincidence of the direction of the minimum radiation of the resonators with the “angles of reflection” of the incident wave, are shown in Fig. 2, c, d.

Thus, the performed studies of the c-functions in the general case make it possible to determine the conditions for the quasi-complete propagation of scattered waves through the gratings of pseudo rotated cylindrical DRs with a fundamental oscillation  $H_{10\delta}$  similar to the Malyuzhinets effect.

It was found that the effect of quasi-complete passage of the incident wave through the DR grating occurs when one of the conditions:

If the direction of the minimum radiation of the resonators coincides with the direction of propagation of the reflected wave.

In the case of vanishing of all the c-functions of the resonators. For the main types of oscillations,  $H_{10\delta}$  the specified condition is realized when one of the equations (2-3) is fulfilled.

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