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ELECTROMAGNETIC WAVES SCATTERING ON COUPLED DIELECTRIC RESONATORS IN THE TIME DOMAIN

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РОЗСІЮВАННЯ ЕЛЕКТРОМАГНІТНИХ ХВИЛЬ НА З'ВЯЗАНИХ ДІЕЛЕКТРИЧНИХ РЕЗОНАТОРАХ У ЧАСОВОЇ ОБЛАСТІ

Розраховані характеристики розсіювання імпульсів на системах зв'язаних діелектричних резонаторів, розташованих в лінії передачи. Виведено система диференціальних рівнянь, яка описує форму розсіяних імпульсів. Знайдено загальне рішення системи. Запропонований підхід дозволяє розраховувати форму розсіюваних імпульсів без застосування функцій Гріну.

Attempts of obtaining the analytical solutions of the Maxwell's equations in the spatial and time domains generally lead to extraordinary cumbersome expressions [1-2]. Finding accurate solutions of Maxwell's equations for complex structures such as the multiple Dielectric Resonator (DR) systems is not possible. A more reasonable to search for new physical laws that facilitate the construction of analytical models of devices with DR and adequately describe these structures. The goal of the current work is the developing of the scattering theory on DR in the time domain.

Consider the problem of scattering of electromagnetic waves $\left(\vec{E}^{+}(\mathrm{t}),\vec{H}^{+}(t)\right)$ on the coupled N DR system. We assume that all DR are made of material with $\tilde{\varepsilon}=\varepsilon_1-i\varepsilon_1''$. We write a general expression for the field $\left(\vec{E}(\mathrm{t}),\vec{H}(\mathrm{t})\right)$ and vth coupled oscillations of the DR system $\vec{\mathrm{e}}^{\mathrm{v}}=\sum_{\mathrm{u}=1}^{\mathrm{N}}\mathrm{b}_{\mathrm{u}}^{\mathrm{v}}\vec{\mathrm{e}}_{\mathrm{u}}$; $\vec{\mathrm{h}}^{\mathrm{v}}=\sum_{\mathrm{u}=1}^{\mathrm{N}}\mathrm{b}_{\mathrm{u}}^{\mathrm{v}}\vec{\mathrm{h}}_{\mathrm{u}}$ [3] ($\mathrm{v}=1,2,...,\mathrm{N}$):

$$div\{[\vec{h}^{\nu*},\vec{E}(t)]+[\vec{H}(t),\vec{\mathrm{e}}^{\nu*}]\}=[\tilde{\varepsilon}\frac{d\vec{E}(t)}{dt}-i\tilde{\omega}^{\nu*}\varepsilon_1\vec{E}(t)]\vec{e}^{\nu*}+\mu_0[\frac{d\vec{H}(t)}{dt}-i\tilde{\omega}^{\nu*}\vec{H}(t)]\vec{\mathrm{h}}^{\nu*}. \quad (1)$$

Where $(\vec{\mathrm{e}}_{\mathrm{u}},\vec{\mathrm{h}}_{\mathrm{u}})$ are known uth oscillations of the partial isolated DR [4].

The solution of the wave $\left(\vec{E}^{+}(\mathrm{t}),\vec{H}^{+}(t)\right)$ scattering on the DR system we will search in the form of expansion on coupling modes of the DR lattice $(\vec{\mathrm{e}}^{\mathrm{v}},\vec{\mathrm{h}}^{\mathrm{v}})$:

$$\vec{E}(t)\approx\vec{E}^{+}(t)+\sum_{s=1}^{N}a^{s}(t)\vec{e}^{s};\ \vec{H}(t)\approx\vec{H}^{+}(t)+\sum_{s=1}^{N}a^{s}(t)\vec{h}^{s}\ . \quad (2)$$

After using the ratios for $(\vec{e}^s, \vec{h}^s)$ and for the $\left(\vec{E}^+(t), \vec{H}^+(t)\right)$ similar (1) after the volume integration of each partial resonator $(\vec{e}_u, \vec{h}_u)$, the equation system with the unknown coefficients $a^s(t)$ has been obtained:

$$\sum_{s=1}^{N}[(2Q_u^D - i)\frac{da^s(t)}{dt} - 2i\tilde{\omega}^s Q_u^D a^s(t)]b_u^s = -\frac{\omega_0}{P_u^D} c_u^+(t)^* \;,\; (u = 1, 2, ..., N), \qquad (3)$$

where the expansion coefficients of the DR field $\left(\vec{e}_u, \vec{h}_u\right)$ on the incident wave $\left(\vec{E}^+(t), \vec{H}^+(t)\right)$ of the transmission line:

$$c_u^{\pm}(t) = -1/2\oint_{s_t}\{[\vec{e}_u, \vec{n}](\vec{H}^{\pm}(t))^* + [\vec{n}, \vec{h}_u](\vec{E}^{\pm}(t))^*\}ds. \qquad (4)$$

$P_u^D = \omega_0\varepsilon''_u / 2\int_{V_u}|\vec{e}_u|^2 dv$ - is the dielectric power loss in the volume of u th resonator: V_u; $Q_u^D = \varepsilon_u/\varepsilon''_u \approx \omega_0 w_u / P_u^D$; $w_u = 1/4\int_{V_n}(\varepsilon_1|\vec{e}_u|^2 + \mu_0|\vec{h}_u|^2)dv$ - is the energy, stored in the dielectric material of the u th resonator.

Provided $Q_u^D \gg 1$, the equation (3) becomes even more simple form:

$$\sum_{s=1}^{N}[\frac{da^s(t)}{dt} - i\tilde{\omega}^s a^s(t)]b_u^s = -\frac{1}{2w_u} c_u^+(t)^*. \qquad (5)$$

A linear system of differential equations (3) or (5), taking into account the expansion (2), gives the general solution of the scattering problem in the time domain.

In the absence of incident waves, solution of (5) has the form of coupled oscillations $a^s(t) = a^s e^{i\tilde{\omega}^s t}$ of the DR system. In the case of incidence of waves falling on the resonators, a general solution of (5) can be obtained in an explicit form:

$$a^s(t) = -\frac{1}{2}e^{i\tilde{\omega}^s t}\int_{-\infty}^{t} e^{-i\tilde{\omega}^s\tau}\frac{\det C_s(\tau)}{\det B}d\tau, \qquad (6)$$

where

$$C_s(t) = \begin{bmatrix} b_1^1 & b_1^2 & ... & c_1^+(t)^*/w_1 & ... & b_1^N \\ b_2^1 & b_2^2 & ... & c_2^+(t)^*/w_2 & ... & b_2^N \\ . & . & ... & . & ... & . \\ b_N^1 & b_N^2 & ... & c_N^+(t)^*/w_N & ... & b_N^N \end{bmatrix}; \quad B = \begin{bmatrix} b_1^1 & b_1^2 & ... & b_1^N \\ b_2^1 & b_2^2 & ... & b_2^N \\ . & . & ... & . \\ b_N^1 & b_N^2 & ... & b_N^N \end{bmatrix}.$$

Vector $[c_v^+(t)^*/w_v]_{v=1,...,N}$ is situated in the s th column of the matrix $C_s(t)$.

It is easy to see that for the coherent harmonic waves the (6) coincides with (6.12), [3]. The fig. 1 show results of the calculation of the rectangular and Gaussian pulses, scattered by the one and two DR.

The obtained analytical solution allows us to directly calculate the shape of the pulses scattered by a system of coupled DR without the use of time-domain Green's functions.

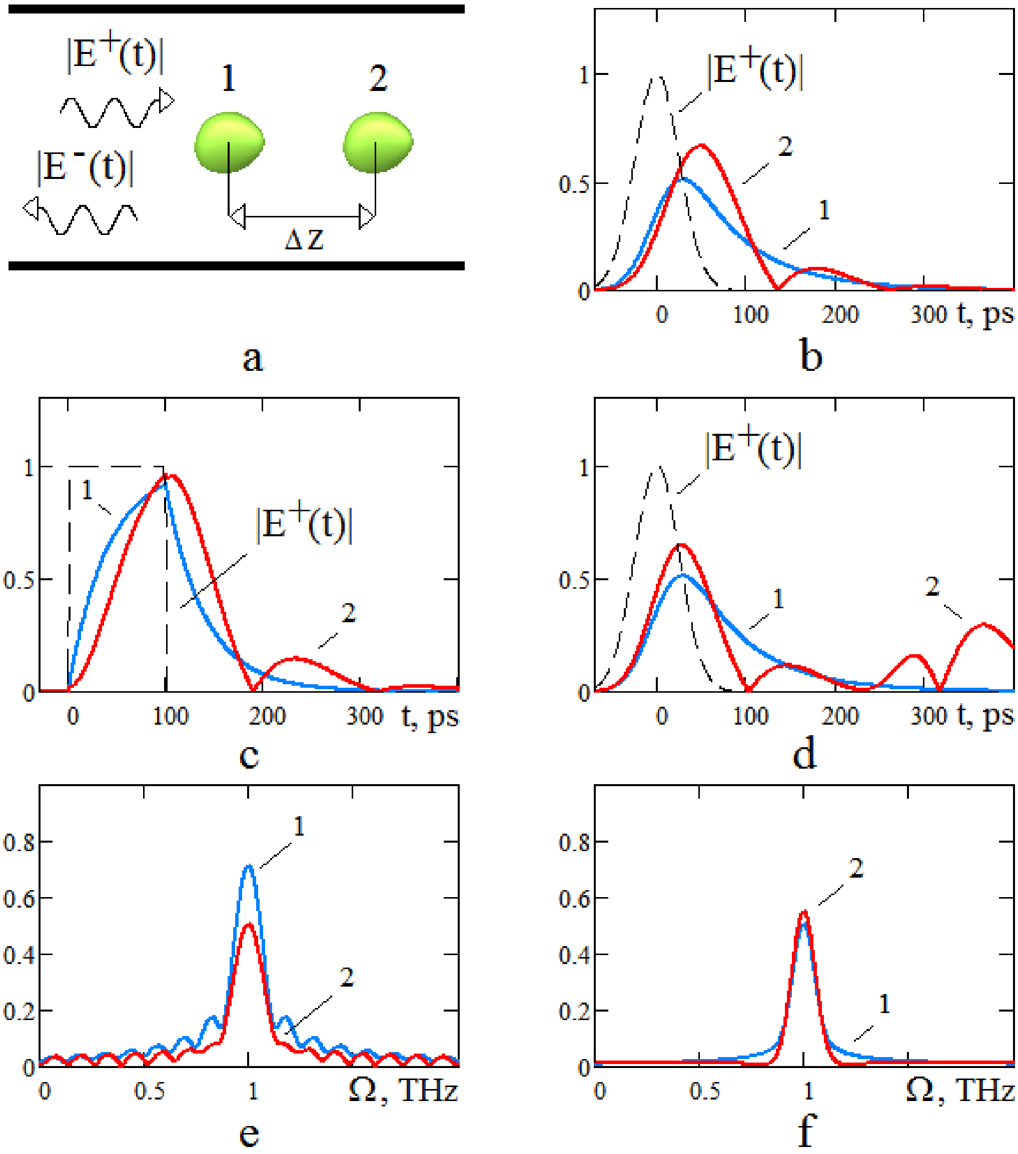

Fig. 1. The pulse envelopes $|E^{-}(t)|$, scattered by the one (1) two (2) DR (b-f) in the transmission line (a); $\tilde{k}_1=\tilde{k}_2=\tilde{k}_{11}^{-+}=0{,}03$; $k_{12}=0{,}02$; $\omega_0=1$THz; Ω - is the frequency of the pulse carrying; $\Gamma\Delta z=\pi/2$ (b, c); $\Gamma\Delta z=181\pi/2$ (c, d).

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